SOFIA UNIVERSITY “ST. KLIMENT OHRIDSKI”

SUMMARY

DOCTORAL DISSERTATION

"Application of the differentiated approach to the mainstream mathematical learning of students with dyscalculia"

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CONTENTS

Introduction

Student performance in Mathematics is influenced by both individual and environmental factors. Specific Learning Disabilities, like dyscalculia, can contribute to difficulties in Mathematics despite normal intelligence and learning opportunities. Cognitive and neuropsychological factors, including perceptual disorders, memory problems, and difficulties with abstract thinking, play a role. Individualized approaches, early diagnosis, and modern technology interventions are important for effective treatment. Addressing mathematical deficiencies is crucial for students with low performance in language courses.

PART ONE: THEORITICAL BACKROUND

CHAPTER 1 - DISCALCULIA

1.1 DEVELOPMENTAL DISORDER

1.1.1 Definition of developmental dyscalculia and its characteristics

Dyscalculia is a learning disability in mathematics, affecting arithmetic, number orientation, and symbol recognition. It can coexist with other learning disabilities but must be differentiated from low performance or other educational needs. Understanding different forms is crucial for appropriate educational approaches. Developmental dyscalculia may co-occur with dyslexia and dysgraphia, as well as other central nervous system disorders. While dyslexia mainly affects reading and writing, dyscalculia specifically refers to math difficulties.

1.1.2. Research approaches to developmental dyscalculia

Research on mathematical difficulties extends beyond dyscalculia, revealing various factors influencing arithmetic performance. Macaruso and Sokol (2022) emphasized comprehensive assessment, identifying psychiatric issues in adolescents initially suspected of dyscalculia. Geary (2000) found deficits in arithmetic across different learning disability groups, indicating broader cognitive impacts. Rourke and Strang (1978) associated left and right hemisphere dysfunction with language-based and non-language-based difficulties. Macaruso and Sokol (2022) highlighted deficits in retrieval, algorithms, and co-occurring language disorders in dyscalculia. Further investigation is needed to understand the complex relationship between learning difficulties, intelligence, and arithmetic performance.

1.2 NEUROPSYCHOLOGY AND MATHEMATICS DISORDERS CAPABILITIES

1.2.1 Neuropsychology

Neuropsychology is a branch of neuroscience that combines psychology with neurology to study the relationship between brain function and human behaviour. While most research in neuropsychology has focused on brain-damaged adults, there has been a recent expansion into studying learning disabilities. Neuropsychologists investigate perceptual, cognitive, and motor deficits in individuals with learning disabilities and their connection to brain structure and function. Clinical neuropsychology specifically focuses on identifying brain dysfunctions that contribute to behavioural disorders. Overall, neuropsychology plays a significant role in understanding the underlying mechanisms of learning disabilities.

1.2.2. A brief review of neuropsychological research on acquired mathematics disorders

Neuropsychological studies have improved understanding of learning disabilities in mathematics, particularly in adults with brain damage. Historical theories and dynamic localization concepts shape research, focusing on cognitive functions like attention, memory, language, and perception.

1.2.3 Neuropsychological classification of acquired disorders of mathematical operations

Research on math operation disorders caused by brain damage has focused on adults with central nervous system damage. The term "acalculia" was introduced to describe the loss of computational abilities. Different types of acalculia were identified, including primary acalculia and acalculia associated with alexia, agraphia, spatial difficulties, or anarithmetria. These categorizations influenced further research on arithmetic difficulties and their relationship with other neurological conditions.

1.2.4. Hemispheric differences in brain function and children's performance in Mathematics

The cerebral hemispheres play distinct roles in language, analytical processing, spatial functions, and holistic perception. Both hemispheres are involved in mathematics learning, with the left focusing on arithmetic and logical analysis and the right shaping perception and problem-solving. Balancing hemispheres enhances education and supports diverse cognitive profiles.

1.3 COGNITIVE AND NEURO-PSYCHOLOGICAL FACTORSRELATED TO LEARNING DIFFICULTIES IN MATHEMATICS

1.3.1 Cognitive and Neuropsychological factors

Learning difficulties in quantities and numbers in young students are influenced by factors like spatial and visual perception, symbol recognition, language ability, memory, cognitive strategies, and metacognition. Students with math learning disabilities often experience information processing issues, such as distraction, perceptual disorders, memory problems, and motor difficulties. Addressing these factors through appropriate strategies can support students with learning disabilities in mathematics. Visual perception, auditory framing, spatial and time organization, and poor visual motor coordination can also contribute to these difficulties.

In addition memory disorders and perceptual deficits can impact students' mathematical abilities. Students with short-term or long-term memory problems require specialized teaching techniques and exercises to improve retention of knowledge. Students with learning disabilities often face challenges in integrating and applying their knowledge in mathematics.

On the other hand, communication disorders, as defined by the APA - DSM V (2013), encompass various conditions such as Language Expression Disorder, Mixed Expression of Language Perception Disorder, Phonological Disorder, Stuttering, and Non-verbal Communication Disorder Otherwise Specified. In these disorders, difficulties in language expression, perception, speech sound production, or speech flow and timing impede academic, occupational, and social functioning. Language Expression Disorder is a developmental condition characterized by difficulties in oral language expression, such as limited vocabulary, grammar errors, and challenges in producing complex sentences. In mathematics, students with this disorder face difficulties in verbal communication, time-constrained tasks, and expressing their understanding of mathematical concepts orally.

Mixed Receptive-Expressive Language Disorder is a developmental condition characterized by difficulties in both language expression and language perception. In mathematics, individuals with this disorder struggle with understanding mathematical terms, comprehending instructions, solving verbal problems, and adapting or solving exercises presented orally. Their challenges in expressing and perceiving language hinder their ability to understand and communicate mathematical concepts effectively.

CHAPTER 2 -EDUCATION OF CHILDREN WITH DYSCALCULIA

2.1. REGULATORY DOCUMENTS

Dyscalculia education is governed by regulatory documents and legislation, including the Individuals with Disabilities Education Act (IDEA) and Section 504 of the Rehabilitation Act. Each state has its own regulations and policies, requiring educators, parents, and policymakers to support their educational needs.

2.1.1 Regulatory Documents in Europe, Greece and Bulgaria

Europe supports inclusive education for children with dyscalculia and learning disabilities through laws, policy documents, and guidelines. The Council and UN Convention provide guidance, with each country having its own regulations. Programs like Erasmus+ fund projects, and the European Parliament encourages member states to promote inclusive practices.

In Greece, laws and policy documents promote equal opportunities for children with dyscalculia and other learning disabilities. Law 3699/2008 emphasizes inclusive education and special education services, while the National Institute of Special Education supports it through professional development and research. In Bulgaria, the Pre-school and School Education Act (PSEA) and the Ministry of Education and Science provide frameworks for special education services. The National Strategy for the Development of Education focuses on improving educational opportunities and providing tailored support for children with dyscalculia.

2.2. INCLUSIVE EDUCATION IN GREECE AND BULGARIA

Inclusive education ensures equal opportunities and quality education for students with dyscalculia. It involves adapting teaching methods and curricula to meet their specific needs. Supported by the UN Convention, inclusive education benefits academic achievement and social development. Collaboration among educators, support staff, and parents is essential. Adequate training, resources, and policies are necessary for successful implementation.

Greece aims to promote inclusive education for students with dyscalculia and other learning disabilities through laws and regulations. Initiatives like Zones of Educational Priority and Individualized Education Plans aim to support schools with high numbers. However, challenges like teacher training and limited resources remain. Addressing these is crucial for effective implementation. Greece implements a differentiated approach for dyscalculia students, addressing unique learning needs through adapted teaching methods, curricula, and assessments. IEPs outline goals, strategies, accommodations, and support services. However, challenges in teacher training and professional development remain. Greece and Bulgaria have implemented differentiated approaches for dyscalculia learners, focusing on IEPs and adapting teaching methods, curricula, and assessments. However, challenges in teacher training and professional development hinder effective implementation. Addressing these issues is crucial for successful differentiated instruction.

CHAPTER 3 - THE ANALYTICAL PROGRAMS AND TEACHING/ LEARNINGOF BASIC MATHEMATICAL CONCEPTS AND PROCEDURES

3.1. THE TEACHING OF FUNDAMENTAL MATHEMATICAL CONCEPTS AND SKILLS

The understanding of mathematics is built on fundamental concepts and skills that are interconnected. If any link in this chain is missing, it can disrupt the overall comprehension. Mathematics uniquely relies on prerequisite knowledge, and neglecting this in teaching can cause learning difficulties and failure for many students. Introducing new concepts without ensuring a solid mastery of prerequisites is ineffective for students with math learning difficulties.

3.3.1. The concept of number

Piaget's theory suggests that children's development of the concept of number progresses through three stages, involving struggles with quantitative comparisons, conflicts between perception and logic, and eventual understanding through reversibility. The concepts of conservation and number typically develop around age 7 or 8, but there is variation among individuals. Recent research highlights the importance of foundational skills like enumeration and estimation in the development of number concepts, which serve as a basis for learning addition and subtraction.

3.3.2. The concept of place value of digits

The decimal numbering system is based on place value and the use of ten digits (0-9). Understanding place value is essential for forming numbers and performing arithmetic operations. Difficulties in grasping place value can be challenging for students. Effective teaching of place value involves using visual representations and a variety of supervisory means, such as analog or non-analogical tools, to enhance students' understanding.

3.3. THE EVOLUTION OF THE PROCEDURES FOR PERFORMING AND MEMORIZING SIMPLE ARITHMETIC OPERATIONS

Research on numerical operations (addition, subtraction, multiplication, and division) highlights the involvement of procedural and declarative knowledge. Addition involves enumeration and recall, while subtraction transitions to declarative methods. Multiplication combines reconstructive and declarative processes. Mental numeracy activities progress through levels, and learning multiplication and division tables simultaneously can enhance understanding. Reducing redundancy and using heuristic rules facilitate learning, while incorporating verb properties aids concept development.

3.4. SOLVING PROBLEMS

Problem-solving is crucial in elementary school mathematics, promoting creative thinking and problem-solving abilities. It involves applying concepts and techniques, but repetitive exercises can overshadow it. Proficiency in numerical operations, mathematical vocabulary, and short-term memory are essential for success. Instruction and practice improve vocabulary, numeracy, and reading abilities.

3.4.1. The stages and strategies of problem solving

G. Polya and Montague proposed problem-solving approaches in mathematics. Polya's approach consists of understanding, planning, executing, and evaluating, emphasizing heuristic strategies. Montague's approach combines cognitive and metacognitive strategies, involving steps like paraphrasing, visualizing, estimating, and checking. Both models emphasize understanding, planning, executing, and evaluating as essential steps in problem-solving, providing systematic frameworks for developing mathematical thinking skills.

PART TWO: RESEARCH APPROACH

CHAPTER 4 - RESEARCH METHODOLOGY

In this chapter of the second part, we will refer in detail to the purpose and sub-objectives of the research, the research questions asked, the selection method and the characteristics of the sample, the instruments, the timetable and the stages of the research and finally the method of statistical analysis of the results.

4.1 PURPOSE AND AIMS OF THE RESEARCH

The research aims to assess the effectiveness of differentiated instruction for students with dyscalculia in mainstream math classes. It has two sub-goals: comparing students with math learning difficulties to those without difficulties to identify psychological and neuro-psychological differences and examining mathematical performance, individual characteristics, and family characteristics between the groups. The research also involves designing, implementing, and evaluating a teaching program within the math curriculum to enhance mathematical understanding for students with learning difficulties

4.2 RESEARCH QUESTIONS

1. How do students with and without learning difficulties in mathematics differ in their performance in mathematical abilities assessments, teacher evaluations, intelligence tests, and learning difficulty tests?
2. What are the individual and family characteristics that distinguish students with learning difficulties in mathematics from those without difficulties?
3. How does the performance of an experimental group receiving a teaching program for students with learning difficulties in mathematics compare to a control group? Is the teaching program effective in improving mathematical performance?

Hypotheses

H1: Differentiated instruction improves mathematical abilities in students with dyscalculia.

H2: Students with dyscalculia exhibit specific cognitive profile.

H3: Individual and family characteristics of students with dyscalculia differ from those without mathematical learning difficulties.

4.3 THE RESEARCH METHODOLOGY

The research aimed to evaluate the effectiveness of a remedial teaching-learning program for students with learning difficulties in Mathematics who were currently studying in an Elementary School. To ensure a valid and objective evaluation, two groups were selected from an initial sample, with efforts made to make them as equivalent as possible. One group was designated as the Experimental Team (ET), while the other was the Control Team (CT). The specific characteristics of the sample, the means of data collection, and the schedule and stages of the research were not provided in the given information.

4.3.1. The original sample and its characteristics

A research study was conducted on elementary school students in Thessaly, where 22 multi-seat schools were randomly selected. Teachers were asked to suggest four 4th grade students per class: two with normal performance and two with low performance in Mathematics, excluding students with various significant issues that could influence the results. The initial sample included 69 students with low performance and 30 students with normal performance in Math. The ratio was based on literature (Butterworth, B. 2005 and Geary D 2002) stating around 6% of students in regular schools have cognitive impairment (dysnumeracy).

4.3.1.1. The selection of Experimental Group and Control Group

The research formed an experimental group (n = 23) and a control group (n = 23) based on a formula considering students' performance in Mathematics, gender, and general intelligence. The groups were intentionally created to have high similarity in key characteristics such as age, gender, cognitive abilities, socio-economic status, and cultural level. The research followed a semi-experimental design with pre-test and post-test assessments in physically equivalent groups. (Kokkinidou, 2020).

4.3.2. The instruments for collecting research data

The research utilized various questionnaires and tests to collect data. Students completed the Wechsler Intelligence Scales for Children (Greek WISC-III) to assess their intelligence, the Athena test for learning difficulties to assess learning difficulties, and criteria for assessing mathematical skills. Teachers provided student performance evaluations and additional teaching support information, while parents filled out questionnaires on individual and social data. The data collection aimed to gather information on intelligence, learning difficulties, mathematical skills, teacher evaluations, and parental input.

4.3.2.1. Psychodiagnostic tools and their usefulness

Individuals exhibit both diatomic differences (variations between individuals) and intra-atomic differences (differences within an individual's development). Studying intra-atomic differences is important for differential diagnosis, particularly in education and learning. Diagnostic-psychological scales like the WISC and Athena tests are commonly used to diagnose learning difficulties and inform tailored teaching programs. However, a comprehensive diagnosis may require a multidisciplinary team and additional tests. Special educators play a crucial role in translating diagnostic information into effective educational strategies.

4.3.2.2. Wechsler intelligence scales for children - The structure and characteristics of the Greek WISC-III

The Greek WISC-III is the Greek version of the Wechsler Intelligence Scale for Children (WISC-III). It assesses various aspects of intelligence in children aged 6-16 through 13 subscales. These sub-scales measure different mental functions, such as memory, abstract thinking, and visual-spatial skills. The test provides scores for verbal intelligence, practical intelligence, and general intelligence. It evaluates intelligence through both acoustic-linguistic and visual-kinetic channels. The test allows for intra-individual comparison, providing insights into a child's strengths and weaknesses and aiding in targeted interventions and support.

4.3.2.3. The usefulness of the Greek WISC-III

The Greek WISC-III has various applications, including psychological evaluations, designing educational programs, assessing developmental potential, determining placement in special education, and conducting clinical and neuropsychological evaluations. It aids in identifying learning difficulties, special abilities, and making decisions for interventions. The individual scales provide insights into cognitive abilities. The test is valuable for diagnosing and treating learning disabilities and cognitive impairments, even though its original purpose was not specifically for neuropsychological evaluation. Gathering information from multiple sources is important for a comprehensive evaluation.

4.3.2.4. Factors - Categories of the WISC subscales

When interpreting learning disability assessments, it's important to group subscales measuring the same skills. Approaches like Wechsler's analysis, Bannatyne categorization, and Horn and Cattell's theory aid in this grouping. Understanding the subscales' measurement of general intelligence (g) helps predict performance deviations. The Wechsler Intelligence Scale for Children (WISC) has four factors: Verbal Comprehension, Freedom from Distraction, Perceptual Organization, and Processing Speed. These Bannatyne-categorized subscales are widely used to assess dyslexic and non-dyslexic children due to their validity and practicality.

4.3.2.5. The characteristics of "Athena" test for diagnosing learning difficulties

The Athena Test is a multidisciplinary diagnostic tool for assessing learning difficulties in children aged 5-9. It evaluates various abilities related to school subjects and provides a detailed diagnostic profile. The test consists of fourteen main tests, including motor, perceptual, mental, and psycholinguistic abilities. It helps identify areas of deficiency and offers insights into the child's thinking process, aiding in tailored interventions and research

Table 1 The scales of the Athena Test in the order of their administration and by sectors of development

I	Mental ability 1. Linguistic proportions 2. Copy Shapes 3. Vocabulary
II	Sequence Memory 4. Number memory Common Sequences (Supplementary) 5. Picture memory 6. Memory of shapes
III	Completion of performances 7. Completion of proposals 8. Completion of words
IV	Graphophonological awareness 9. Distinguish graphs 10. Distinguish sounds 11. Composition of sounds
V	Neuro-psychological maturity 12. Visual-motor coordination 13. "Right-left" perception 14. Flipping

4.3.2.6. The questionnaire of individual developmental data and family and social data of the student

When diagnosing learning difficulties, gathering information from various sources is crucial. This includes the child's developmental history, family context, physical health, psychological development, and social adaptation. Interviews, questionnaires, and assessments are commonly used to collect data. Evaluating all these areas, including the family environment, is essential for an accurate diagnosis of learning difficulties in mathematics.

4.3.2.7. Student performance evaluation sheet

In the absence of weighted tests, teachers' assessment of students' school performance is considered significant. Researchers emphasize the importance of teachers' evaluations in measuring academic achievements. The evaluation process involves teachers answering questions about various aspects of school performance, including general performance, mathematics, and language skills. The assessment considers the specific curriculum and course attended by the students. Despite being subjective, teacher evaluations provide valuable information, helping determine each student's deviation from the class average.

4.3.2.8. The criteria for evaluating Mathematical skills

In the absence of a weighted instrument, researchers developed their own assessment criteria to evaluate the performance of 5th-grade students in Mathematics. The criteria were formulated based on insights from neuropsychological, cognitive neuropsychological research, and Mathematics education studies. The criteria encompassed understanding of numbers, the decimal numbering system, number representation, execution of arithmetic operations, and problem-solving skills. Multiple versions of the criteria were created to ensure fairness and reliability in assessing students' mathematical abilities.

4.3.2.9 The scores of the criteria

Students' performance on the three equivalent grading criteria was assessed based on their correct answers on each test. The score for each test was determined by the number of individual tests and their difficulty level. The total score for each criterion ranged from 0 to 80 points. To facilitate statistical processing, the scale was converted to a grade scale, as the evaluation of students' performance in upper primary school grades is typically done using a ten-digit numerical scale.

Part I

The Mathematics assessment criterion included multiple individual tests with specific scoring methods. Tests A1, A2a, A2b, A2c, and A3 were scored based on the number of mistakes, with different point values assigned for different error ranges. Tests B1, B2, and B3 awarded points for correctly written numbers from a given set. Test C1 awarded points for each correct answer, while Tests C2, C3, C4, and C5 awarded points for each correctly circled digit. Test C6 consisted of individual tests, each worth 1 point. The total score for the first part of the criterion was 30 points, divided into three parts: A, B, and C, each worth 10 points.

Part II

The second part of the Mathematics assessment criterion focused on arithmetic operations and had a total score of 30 points. Test A did not have numerical scoring for operation symbols. Test B1 earned 0.3 points for each correct answer in simple addition and subtraction, totalling 3 points. Test B2 earned 0.5 points for each correct answer in simple multiplication and division, totalling 5 points. Test B had an overall score of 8 points. Test C1 earned 0.5 points for each individual test, totalling 2 points. Test C2 had varying scores based on algorithm complexity, ranging from 1 point to 2 points, resulting in a total score of 22 points for test C. The second part of the criterion had a total score of 30 points, with part B worth 8 points and part C worth 22 points.

Part III

The third part of the Mathematics assessment criterion focused on problem-solving and had a total score of 20 points. The first ten simple problems earned 1.5 points each, totalling 15 points. Complex problem one and complex problem two were each worth 2.5 points. The scoring for the complex problems involved awarding points for correct choices of operations and additional points for correct final results. The third part of the criterion had a total score of 20 points, with 15 points for the simple problems and 2.5 points each for the complex problems.

4.4 THE SCHEDULE AND STAGES OF THE RESEARCH

The research consisted of two phases:

- Formation of the initial sample: Children with learning difficulties in Mathematics and children without difficulties were selected.
- Formation of experimental and control groups: Students with difficulties were divided into an experimental group and a control group. The didactic intervention was then carried out exclusively with the students in the experimental group.

4.4.1. Preliminary research

During the second semester of the academic year 2022-23, preparations for the research were conducted, including the development of the Mathematical skills criterion and the questionnaire for data collection. Research tools were acquired and studied systematically. A preliminary phase involved 12 students from three primary schools in Heraklion, working alongside a committee and experts. Versions of the criterion were created, adjustments were made, and research permissions were obtained. The research was planned to be conducted in 64 primary schools in the Thessaly region.

4.4.2. The collection and evaluation of the first data

Data collection began in the first semester of the 2022-23 academic year, involving visits to schools and teacher participation. Students with learning difficulties and those with normal performance were paired based on similar characteristics. Detailed instructions were given to teachers, and individual assessments were conducted with a total of 69 students with learning difficulties and 30 students with normal performance. Intra-individual profiles were created, analysing mental abilities and identifying strengths and weaknesses. Pre-testing of mathematical knowledge and evaluation of assessment criteria were conducted.

4.4.3. The general planning of the didactic intervention

The teaching intervention for the research project began in January 2023 and lasted until May of the same year. The researcher visited schools daily, following a fixed program and dedicating 16 hours per week to teaching. Prior consultations were held with school principals, counsellors, and parents. Students were organized into co-located school groups, with teaching sessions lasting two hours. Detailed information on the composition of the Experimental and Control groups, including gender distribution, general intelligence index, and performance in the math skills criterion (pretest), was recorded for future statistical comparisons between the groups.

4.5 THE METHOD OF STATISTICAL ANALYSIS OF DATA EXPERIMENTAL TEACHING

SPSS was used for statistical analysis of empirical data in the study. Variables were categorized as nominal and quantitative. Descriptive statistics, chi-square, Fisher's Exact Test, independent samples t-tests, dependent samples t-tests, and Pearson's correlation coefficient were utilized. Significance level of $p = .05$ was applied to all tests.

4.5.1 Introduction

Part B of the research involved designing, implementing, and evaluating a mathematics curriculum with objectives focused on creating a conducive learning environment, developing mental functions, acquiring mathematical concepts and skills, cultivating mathematical language, and enhancing cognitive and metacognitive strategies. Teaching methods included teamwork and cooperative techniques, while assessment procedures involved tests and qualitative analysis of errors. The curriculum aimed to optimize learning, develop mathematical abilities, and utilize assessments to support student progress.

4.5.2 Introductory lesson The teaching of simple addition operations and subtraction - mental calculations (2 hours)

The objectives of the curriculum include building a rapport with the teacher, developing a solid understanding of addition and subtraction as reverse operations, performing simple addition and subtraction using various methods and representations, reciting the number sequence, adding up to four one-digit numbers, analysing two-digit numbers, and mentally performing addition and subtraction of two-digit numbers with and without regrouping.

4.5.2.1. The necessity of teaching the simple operations of addition, of subtraction and mental calculations

The analysis of errors in addition and subtraction revealed that students often relied on immature and time-consuming strategies. Learning disabilities affected straight and inverse counting at specific numbers. Researchers suggest targeted exercises to improve arithmetic operations. Some students struggled with understanding and using operation symbols, which may be related to perceptual and cognitive disorders. The didactic treatment addressed these difficulties, but further investigation is needed.

4.5.2.2. The teaching course of the first lesson

To develop an understanding of addition and subtraction, the teaching researcher utilized various problem types and real-life situations. Different tools and models were employed, and alternative methods of performing operations were introduced. The transpositional property and the neutral element of addition were emphasized. Visual aids such as the number line and manipulative materials were used to support learning. Short-term techniques like Pre-arranged Oral Response and Exercise Cards were used for processing and repetition. The three phases of practice tabs focused on memory and oral questioning.

4.5.2.3. Critique of teaching

In the initial teaching intervention, students with learning difficulties in the Experimental Group had varying performance levels. Individualized goal-setting and assessment of thinking processes were implemented. Inefficient strategies were identified, and alternative strategies were taught. Practice and repetition focused on simple operations and mental calculations. Short-term techniques were used to support learning during repetitions.

4.5.3 Lesson two: Teaching the simple operations of multiplication and division (2 hours)

The objectives for students in relation to multiplication and division are to:

- Understand multiplication as repetitive addition and the properties associated with it.
- Learn the procedures for multiplication and division, both orally and in writing.
- Solve division problems using empirical methods and understand the concept of division as the reversal of multiplication.
- Recognize multiplication and division as inverse operations.

4.5.3.1. The need to teach the simple operations of multiplication and division – teaching method

In teaching multiplication to students with learning difficulties, a modified version of Graham's method was employed. This approach focused on specific products, utilized the properties of transposition, and emphasized memorization of multiplication tables for 0, 1, and 10. By reducing the number of products to be learned and introducing heuristic rules, the aim was to simplify the learning process and facilitate understanding for students.

4.5.3.2. The teaching course of the second course

To teach multiplication and division, various approaches were used, including grouping and layout methods. Real materials and problematic situations were employed for conceptual understanding. Division was introduced through procedures like repeated subtraction. Memorization of multiplication tables was emphasized, using practical techniques and two-sided practice cards. The teaching approach aimed to combine experiential learning with procedural knowledge for better understanding and memorization.

4.5.3.3. Critique of teaching

Students faced challenges in learning multiplication, especially memorizing individual products and the nine multiplication table. Teaching methods involving practice cards and finger techniques were effective in engaging students. Memory sequence difficulties posed additional challenges. Division operations were relatively easier for students who mastered multiplication. Subsequent lessons provided additional practice and evaluation using repetition, consolidation exercises, practice cards, and cooperative techniques to address these challenges.

4.5.4 Lesson three: The position value of the digits (2 hours)

The objectives for students in this context are to develop an understanding of the position value of digits in multi-digit numbers, learn how to write the decimal expansion of a natural number, and gain the ability to read numbers in various formats.

4.5.4.1. The need to teach the decimal number system and of the position value of the digits

Research suggests that students, especially those with learning difficulties in mathematics, encounter challenges in understanding the decimal numbering system and the significance of digit position. The understanding of positional value is essential for comprehending numbers, making comparisons, performing mathematical operations, and effectively writing and reading numbers.

4.5.4.2. The teaching course of the third course

To facilitate the understanding of the decimal numbering system and the value of digit position, the teaching researcher employed polybasic materials such as Dienes cubes. Through inductive reasoning and questioning, students learned about groupings and exchanges in the decimal system. The role of zero and the significance of each digit's position were emphasized, and examples were provided using various materials. Decimal expansions of multi-digit numbers and alternative methods of reading numbers were introduced. Hands-on exercises were conducted, following a cognitive apprenticeship teaching approach, where students worked individually or in pairs to reinforce their understanding.

4.5.4.3. Critique of teaching

The use of supervisory tools, including microcomputers and materials like Dienes cubes, helped students in the Experimental Group understand the concepts of grouping and exchange in the decimal numbering system. Through examples and hands-on activities, students gained an understanding of the structure of the decimal number system and the positional value of digits. The two-way relationship between symbols and materials was emphasized, where students practiced grouping materials according to the decimal system and writing corresponding numbers, as well as representing numbers using materials or illustrative means.

4.5.5 Lesson four: Layout and comparison of numbers (2 hours)

The objectives for students in this context include: comparing integers and using comparison symbols correctly, ordering integers from smallest to largest and vice versa, placing integers on a number line, and inserting one or more integers between two given integers. These objectives aim to develop students' understanding of integer comparison, ordering, and placement on a number line, as well as their ability to work with and manipulate integers effectively.

4.5.5.1. The need to teach comparison and ordering numbers

The ability to compare and order numbers is closely linked to understanding the meaning of numbers. It serves as a prerequisite for comprehending the positional value of digits, understanding concepts like "before" and "after," and using comparison symbols such as greater than (>) and less than (<). Comparing two-digit and three-digit numbers with the aid of supervisory material has proven to be especially beneficial in developing this skill (Gangon, J 2005).

4.5.5.2. The course of teaching the fourth lesson

The teacher utilized multi-base material and relevant questions to teach students how to compare and order two-digit and three-digit numbers. They induced two conclusions: (I) When comparing numbers with different digit counts, the number with more digits is larger. (II) When comparing numbers with the same digit count, the leftmost digits are compared first, followed by subsequent digits until a difference is found. Examples and exercises were provided to illustrate the ordering of numbers, including cases with larger numbers and numbers containing zeros. Students actively participated in individual and paired work, following the cognitive apprenticeship teaching approach.

4.5.5.3. Critique of teaching

Students with difficulties in number comparison may struggle due to challenges in abstract reasoning, visual discrimination, visual correlation, spatial organization, and visual memory. Despite systematic teaching, these students may still make mistakes, albeit to a limited extent. To support their learning, metacognitive questions such as "how did you make the comparison" or "how did you perform the layout" were found to be beneficial. By asking these questions, students were able to reflect on their thought processes and often self-correct their mistakes.

4.5.6. Lesson five: Reading and writing multi-digit numbers (2 hours)

The objectives of the curriculum are for students to:

- Identify and spell integers up to 1,000,000,000.
- Establish a connection between verbal and symbolic representations of numbers and be able to transition between the two.
- Understand the rules of oral numbering and count large numbers using various methods.

4.5.6.1. The need to teach reading and writing numbers

Developing the ability to read and write multi-digit numbers is important in understanding and working with larger numbers. The use of multi-base supervisory material aids in this skill development. However, students may face challenges in correctly writing multi-digit numbers, often due to difficulties in syntactic processing of digits.

4.5.6.2. The teaching course of the fifth lesson

The teacher utilized polybasic material to teach students how to write and read three-digit numbers. They also introduced four-digit numbers and emphasized the role of zero and the dot in indicating the position of digits. Students practiced writing and reading larger multi-digit numbers by dividing them into three-digit parts. This approach facilitated the development of their skills in handling multi-digit numbers.

Through examples and exercises, the following concepts were understood by the students:

The teaching approach focused on understanding the composition of multi-digit numbers and the role of each digit in different classes. Students learned that each class consists of units, tens, and hundreds, with the first class potentially having one or two digits. Zero can appear in any position except the first position from the left. Dots were used to separate three-digit segments representing thousands, millions, billions, etc. Through writing and reading exercises, students practiced applying these concepts and developed their skills in handling multi-digit numbers.

4.5.6.3. Critique of teaching

Through the use of supervisory tools and a step-by-step approach, students gained an understanding of writing and reading multi-digit numbers. Initially, some students made mistakes in omitting or reducing zeros, but with support from classmates or the teacher, they corrected their errors. This process highlighted their improving ability to accurately write and read multi-digit numbers.

4.5.7 Lesson six: The act of addition (2 hours)

The objectives for students in learning addition include understanding the concept of addition, mastering the addition algorithm both horizontally and vertically, utilizing a carrying method for vertical addition, transitioning from horizontal to vertical addition, comprehending the properties of addition (commutative, associative, and identity), performing addition with multiple additives up to four, and verifying addition results. The focus is on developing a solid understanding of addition operations and applying them accurately and efficiently.

4.5.7.1. The necessity of teaching the act of addition

Despite the fact that students with learning difficulties in Mathematics had a satisfactory performance in addition algorithms, it was deemed necessary to teach the relevant concept and its algorithm with integers and to place special emphasis on their understanding, in addition to mechanical execution which to a large extent cultivates school and textbooks.

4.5.7.2. The course of teaching the sixth lesson

The teaching approach for introducing addition operations and algorithms involved assessing students' understanding of simple addition, utilizing manipulative materials and virtual representations, and gradually transitioning to symbolic representations. Real-life experiences and visualizations were used to explain the concept of carrying over. The execution of addition algorithms was demonstrated both horizontally and vertically, with an emphasis on understanding and verification. The students actively engaged with manipulative materials and solved exercises individually and in pairs, following a cognitive apprenticeship teaching approach.

4.5.7.3. Critique of teaching

Students with learning difficulties in Mathematics of Experimental Group did not encounter significant difficulties with the concept and algorithm of addition. The biggest difficulties were in the additions with more than two additives and various techniques were proposed to deal with them.

4.5.8 Lesson seven: The act of subtraction (4 hours)

The teaching objectives for subtraction include understanding its various interpretations, converting horizontal to vertical format, executing algorithms without and with borrowing, and verifying results. The approach involves explanations, demonstrations, and engaging exercises to enhance understanding and application of subtraction operations.

4.5.8.1. The necessity of teaching the act of subtraction

Subtraction is considered more challenging than addition, especially when using the algorithm with borrowing. Two methods exist for executing this algorithm: genuine borrowing and adding the same number to both minuend and subtrahend. In the teaching intervention, one method was chosen to avoid confusion and build upon students' existing knowledge. Understanding positional value and basic subtraction operations is crucial for effectively executing the subtraction algorithm.

4.5.8.2. The course of teaching the seventh lesson.

The teaching approach for subtraction involved assessing students' understanding, emphasizing restructuring the minuend, and introducing the subtraction algorithm through problem-solving. Mathematical vocabulary, column relationships, and mental connections were highlighted. Exercises addressed errors and misconceptions, promoting active engagement and cognitive apprenticeship.

4.6.8.3. Critique of teaching

The students of Experimental Group with learning difficulties encountered particular difficulties, in the subtraction algorithm and especially in subtractions with repetitive "borrowings". Their most common systematic mistakes were subtracting the smallest from the largest regardless of their position and forgetting to give or donate a tenth to the deductible. The technique of eloquent thinking especially helped to avoid mistakes. This technique also revealed the systematic mistakes of the students and helped to reduce their cognitive impulsivity.

4.5.9 Lesson eight: Addition and subtraction problems (2 hours)

The teaching intervention aimed to develop problem-solving skills in addition and subtraction. Students were guided to solve simple and complex problems, understand data and requirements, and employ effective problem-solving strategies. Collaboration and sharing of strategies were encouraged, along with breaking down complex problems into simpler ones. The approach emphasized critical thinking and analytical skills in problem solving.

4.5.9.1. The need to teach prosthetic problems

Problem-solving activities in mathematics enhance practical application and motivation to learn. Systematic teaching and integration across the curriculum support concept introduction, skill development, and knowledge reinforcement. Students with cognitive disabilities require specialized strategies to address difficulties in solving verbal problems. Poor performance in word problems emphasizes the need for targeted interventions.

4.5.9.2. The course of teaching the eighth lesson

The teacher-researcher employed prosthetic problems categorized by Carpenter to teach problem-solving strategies. Specific strategies were taught for each phase, including reading, data analysis, representation, evaluation, and problem breakdown. The role of keywords was emphasized, and students solved problems in pairs and individually using a cognitive apprenticeship approach.

4.5.9.3. Critique of teaching

The teaching approach resulted in a significant improvement in the students' performance in prosthetic problems. The students showed the least improvement in simulation problems, which are not commonly found in school textbooks. The students were heavily influenced by keywords, and despite the teacher's remarks and clarifications during teaching, keywords continued to be a source of errors for some students.

4.5.10 Lesson nine: The act of multiplication (4 hours)

The objectives for students in multiplication include understanding it as repetitive addition, applying the distributive property, knowing the properties of multiplication, mastering various multiplication algorithms, and being familiar with multiplication tests. The focus is on developing a conceptual understanding of multiplication, applying it in different scenarios, and being able to perform multiplication operations efficiently and accurately.

4.5.10.1. The necessity of teaching the act of multiplication

The multiplication algorithm presents challenges for students, including difficulties with memorizing multiplication tables and managing the complexities of the algorithm. These complexities include alternating between multiplications and additions, handling two-digit numbers, transferring numbers across columns, and ensuring correct digit placement. Students with Mathematics difficulties often struggle with these requirements and lack a comprehensive understanding of the algorithm, leading to systematic errors.

4.5.10.2. The teaching course of the ninth lesson

The teaching process for multiplication involved reinforcing basic multiplication operations and using problematic situations to enhance understanding. Material and figurative representations were used to clarify concepts, and algorithms for various multiplication scenarios were demonstrated. Incorrect algorithms were presented for error identification and correction. Students engaged in exercises individually and in pairs, following a cognitive apprenticeship approach.

4.5.10.3. Critique of teaching

After systematic teaching, monitoring, and repetition, most students successfully learned and mastered the multiplication algorithm. However, students with lower practical intelligence and visual-spatial difficulties struggled with systematic errors, such as incorrect placement of partial products and mistakes in carrying digits. These errors were attributed to a lack of understanding or incorrect recall of basic concepts. Some students who lacked knowledge of simple operations either did not attempt the algorithm or made random attempts to solve it.

4.5.11 Lesson ten: The act of division (4 hours)

The objectives for students in division include understanding the concept of division and differentiating between measurement division and division states. They should be able to perform division algorithms with one-digit and two-digit divisors without remainder, known as perfect division. Students should also understand the inverse relationship between multiplication and division and be able to verify division results through testing.

4.5.11.1. The necessity of teaching the act of division

The division algorithm is considered the most challenging among arithmetic algorithms due to its complex nature and the requirement of a strong foundation in addition, subtraction, and multiplication. It also demands visual-spatial perception and involves standard verbal expressions that may cause confusion without explaining the algorithm's meaning. Additionally, division has a dual interpretation as both sharing and measurement. As a result, students with learning difficulties in Mathematics often struggle with division, leading to errors and difficulties (Agaliotis, 2000).

4.5.11.2. The course of teaching the tenth lesson

The teaching process for division began with sharing and measurement divisions using manipulative materials and visual representations. The standard division algorithm was introduced gradually, starting with one-digit divisors and advancing to more complex cases. Students actively engaged in practicing examples and correcting errors. The cognitive apprenticeship teaching approach was followed, allowing for individual and paired work.

4.5.11.3. Critique of teaching

Students with learning difficulties in Mathematics faced challenges in division, particularly when dealing with two-digit divisors. Determining the quotient in such cases was a significant difficulty. Some students relied on repetitive multiplication with single-digit numbers instead of utilizing rounding or mental calculation techniques. Limited teaching and practice time may have contributed to unsatisfactory performance in the meta test. Visual-spatial difficulties and sequence memory problems were identified as additional obstacles. Initial performance in division practice varied, with some students demonstrating a lack of understanding of the algorithm.

4.5.12 Lesson eleven: Multiplication and division problems (2 hours)

The objectives for students in mathematics problem-solving include:

- Solving simple and complex multiplication and division problems.
- Identifying the data and requirements of the problem.
- Developing strategies to represent problem data effectively.
- Presenting problem-solving strategies and sharing answers with classmates.
- Breaking down complex problems into simpler components and solving them separately.

4.5.12.1. The need to teach multiplication problems type

Teaching models for multiplication and division align with students' abilities and experiences. Students with math difficulties may need additional strategies. Pretest results revealed poor performance in multiplication word problems.

4.5.12.2. The teaching course of the eleventh lesson

Vergnaud categorization and visualization techniques were used to teach multiplicative problems. Breaking down complex problems and solving them individually were suggested strategies. Cognitive apprenticeship teaching scheme was employed for students to practice multiplication problem-solving.

4.5.12.3. Critique of teaching

The didactic intervention resulted in improved performance in multiplication problems, while comparison and division problems presented more challenges. Students were influenced by keywords, but visualization techniques aided understanding. Teaching division problems of measurement was successful, and students in the experimental group successfully solved complex multiplication and prosthetic problems.

CHAPTER 5 - RESULTS

5.1 COMPARISONS OF STUDENTS 'PERFORMANCE WITH DIFFICULTIES MATHEMATICS WITH STUDENTS WITHOUT DIFFICULTIES IN CRITERIA FOR MATHEMATICAL COMPETENCES AND IN PSYCHOLOGICAL TEST

5.1.1 Introduction

The first chapter of the research compares students with and without learning difficulties in Mathematics, examining psychological and neuropsychological development, mathematical skills, and academic performance. The WISC and Athena tests, along with teacher evaluations, will be used to assess the differences between the two groups. A grading scale of 1-10 will be employed for the assessments.

5.1.2 Comparisons of the performance of the students of the two groups in the criterion of assessment of mathematical abilities (pretest)

A comparison between students with learning difficulties in Mathematics and students with normal performance revealed significant differences. Students with normal performance outperformed those with learning difficulties in all assessed areas, particularly in arithmetic operations, problem-solving, and positional value understanding. Students with normal performance also displayed more consistent overall performance, except in problem-solving where variability was higher.

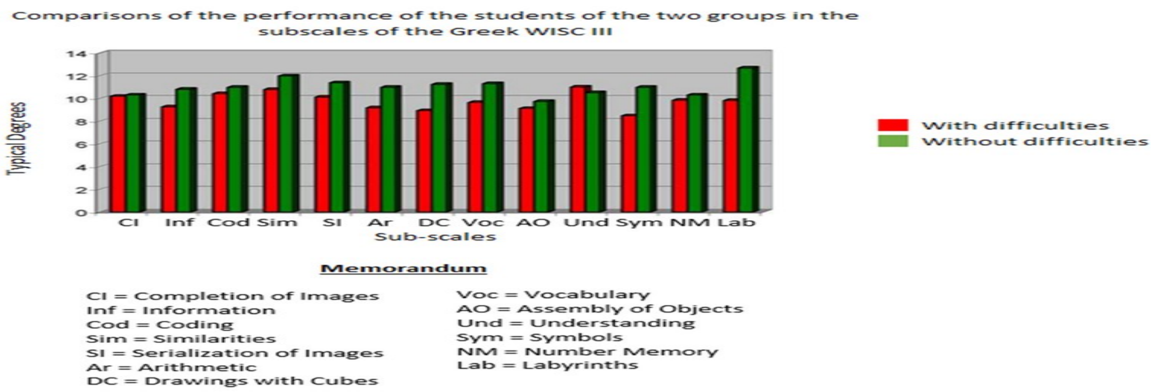
5.1.3 Comparisons of the school performance of the students of the two groups, based on the evaluation of their teachers

A comparison between students with learning difficulties in Mathematics and students with normal performance, based on teacher evaluations, revealed significant differences in school performance. Students without difficulties outperformed those with difficulties in all areas, including general performance, Mathematics, reading, spelling ability, written expression, and oral comprehension. The differences were statistically significant, with the largest disparity observed in Mathematics. The group with difficulties also displayed greater variability in cognitive areas assessed by teachers.

5.1.4 Comparisons of the performance of the students of the two groups in the Wechsler intelligence scales of the Greek WISC-III

A comparison between students with learning difficulties in Mathematics and their peers with normal performance using the Greek WISC-III revealed significant differences in general intelligence indicators and individual scales. The largest disparities were observed in practical scales such as Labyrinths, Symbols, and Drawings with Cubes, as well as in word scales like Numerical, Vocabulary, and Information. Graphical representation illustrated the notable differences between the two groups.

Graph 1The average performance of the WISC subscales of the group of students with learning difficulties in Mathematics and the group of their classmates without difficulties



5.1.5 Comparisons of the performance of the students of the two groups in the scales of Athena test of diagnosis of learning difficulties

A comparison between students with learning difficulties in Mathematics and their peers without difficulties using individual tests from the Athena test revealed significant differences in Vocabulary, Completion of sentences, Completion of words, Memory of numbers, and Distinction of graphs. Students with learning difficulties showed poorer performance in visual-motor coordination and memory of common sequences and counting tests. No significant differences were found in Right-Left Perception or limb preference.

Graph 2The average performance on the Athena scales test of the group of students with learningdifficulties in Mathematics and the group of their classmates without difficulties

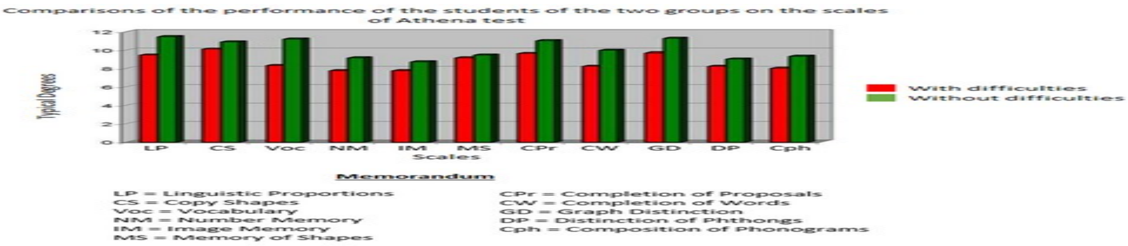


Table 2Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of their visual-motor coordination.

STUDENT CATEGORY	**VISUAL-KINETIC COORDINATION**		
	SUFFICIENT	**INCOMPLETE**	**TOTAL**
STUDENTS WITH DIFFICULTIES	**20** (55,6%)	**16** (44,4%)	**36** 100%
STUDENTS WITHOUT DIFFICULTIES	**29** (96,7%)	**1** (3,3%)	**30** 100%
TOTAL	**49 74,2%**	**17 25,8%**	**100 100%**

Statistical significance check: x^2= 14,462 df= 1 p=,000

Statistically significant differences ($p < 0.001$) were found in visual-motor coordination between students with and without learning difficulties in Mathematics. Among students with learning difficulties, 44% exhibited poor visual-motor coordination, while only one student (3.3%) without learning difficulties showed poor visual-motor coordination. The Athena test evaluates the ability to perform precise hand movements based on visual sensory information.

Table 3Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of the concept of "Right-Left".

STUDENT CATEGORY	**PERCEPTION "RIGHT-LEFT"**		
	SUFFICIENT	**INCOMPLETE**	**TOTAL**
STUDENTS WITH DIFFICULTIES	**29** (80,6%)	**7** (19,4%)	**36** 100%
STUDENTS WITHOUT DIFFICULTIES	**26** (86,7%)	**4** (13,3%)	**30** 100%
TOTAL	**55 83,3%**	**11 16,7%**	**55 83,3%**

Statistical significance check: x^2= 0,440 df= 1 p=,507

No statistically significant differences were found in Right-Left Perception between students with and without learning difficulties in Mathematics. The Right-Left Perception scale assesses the ability to distinguish between the left and right sides of the body. The Pleurisy scale test examines whether a child has a clear preference for one side of the body or if they have an undifferentiated preference across both sides in terms of foot, hand, eye, and ear.

Table 4 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of hand ribbing.

STUDENT CATEGORY	LATERALIZATION (HAND)			
	RIGHT SIDE	LEFT SIDE	DIFFERENTIATED	TOTAL
STUDENTS WITH DIFFICULTIES	32 88,9%	0 0%	4 11,1%	36 100%
STUDENTS WITHOUT DIFFICULTIES	27 90%	2 6,7%	1 3,3%	30 100%
TOTAL	59 (89,4%)	2 (3%)	5 (7,6%)	66 (100%)

Statistical significance check: x^2= 3,709 df= 2p=,157

Table 5Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of flanking for the foot.

STUDENT CATEGORY	LATERALIZATION (FOOT)			
	RIGHT SIDE	LEFT SIDE	DIFFERENTIATED	TOTAL
STUDENTS WITH DIFFICULTIES	16 44,4%	11 30,6%	9 25%	36 100%
STUDENTS WITHOUT DIFFICULTIES	9 30%	13 43,3%	8 26,7%	30 100%
TOTAL	25 (37,9%)	24 (36,4%)	17 (25,8%)	66 (100%)

Statistical significance check: x^2= 1,654 df= 2p=,437

Table6 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of flanking for the eye.

STUDENT CATEGORY	LATERALIZATION (EYE)			
	RIGHT SIDE	LEFT SIDE	DIFFERENTIATED	TOTAL
STUDENTS WITH DIFFICULTIES	23 63,9%	10 27,8	3 8,3%	36 100%
STUDENTS WITHOUT DIFFICULTIES	18 60%	6 20%	6 20%	30 100%
TOTAL	41 (62,1%)	16 (24,2%)	9 (13,6%)	66 (100%)

Statistical significance check: x^2= 2,082 df= 2p=,353

Table 7 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of flanking for the ear.

STUDENT CATEGORY	LATERALIZATION (EAR)			
	RIGHT SIDE	LEFT SIDE	DIFFERENTIATED	TOTAL
STUDENTS WITH DIFFICULTIES	10 27,8%	12 33,3%	14 38,9%	36 100%
STUDENTS WITHOUT DIFFICULTIES	16 53,3%	5 16,7%	9 30%	30 100%
TOTAL	26 (39,4%)	17 (25,8%)	23 (34,8%)	66 (100%)

Statistical significance check: x^2= 4,849 df= 2p=,089

Table 8 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of their performance in common sequences (days - months).

STUDENT CATEGORY	COMMON FOLLOWS (DAYS - MONTHS)		
	SUFFICIENT	INCOMPLETE	TOTAL
STUDENTS WITH DIFFICULTIES	17 (47,2%)	19 (52,8%)	36 100%
STUDENTS WITHOUT DIFFICULTIES	25 (83,3%)	5 (16,7%)	30 100%
TOTAL	42 63,6%	24 36,4%	66 100%

Statistical significance check: x^2= 9,221 df= 1 p=,002

Table 9 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of their performance in common sequences (numbering).

STUDENT CATEGORY	COMMON FOLLOWS (NUMBERING)		
	SUFFICIENT	INCOMPLETE	TOTAL
STUDENTS WITH DIFFICULTIES	17 (75%)	9 (25%)	36 100%
STUDENTS WITHOUT DIFFICULTIES	30 (100%)	0 (0%)	30 100%
TOTAL	57 86,4%	9 13,6%	66 100%

Statistical significance check: F.E.T. p=,003 df= 1 p=,002

5.1.6 The correlation of the three IQs of the WISC with the mental capacity scales of the Athena test

Significant positive correlations were found between intelligence quotients (general, verbal, practical) and linguistic abilities for both students with and without mathematics difficulties. The correlations indicated a relationship between intelligence and language skills, with slightly higher values observed in students without difficulties. The correlation between verbal and practical intelligence varied between the two groups.

5.1.7 The correlation of the three IQs of the WISC with the performance of the students of the two groups in the mathematical competence criterion (pretest) and its parts

Correlations between intelligence quotients and mathematical performance were examined for students with and without math difficulties. For students with difficulties, significant correlations were found between pretest performance and specific subtests, but not with intelligence quotients. In contrast, students without difficulties showed significant correlations between performance, subtests, and intelligence quotients. The correlations were stronger for students without difficulties, indicating a stronger relationship between intelligence and mathematical performance in this group.

5.2 COMPARATIVE STUDY OF INDIVIDUAL AND FAMILY CHARACTERISTICS OF STUDENTS WITH DIFFICULTIES AND WITHOUT DIFFICULTIES IN MATHEMATICS

5.2.1 Introduction

A comparative study was conducted to diagnose learning disabilities and assess children comprehensively. It involved a large group of students with math difficulties and their classmates without difficulties. Information was collected through questionnaires completed by parents, focusing on developmental history, family background, and individual characteristics. The completion rates were similar for both groups, with significant contributions from mothers. The high participation of mothers ensured the validity and reliability of the obtained answers.

5.2.2 The family characteristics of the students of the two groups (difficulties - without difficulties)

This section presents a comparative analysis of the family characteristics of students in two groups: those with math difficulties and their classmates without difficulties. The comparison includes demographic characteristics, parents' attitudes towards mathematics, ambition level for their children, parental assistance with school lessons, assessments of children's difficulties, parental difficulties in school subjects, and assessments of children's hyperactivity and distractibility.

5.2.2.1 Comparisons of the demographic characteristics of the families of the two groups

A comparative analysis of family characteristics was conducted between students with math difficulties and those without difficulties. The study found no significant differences in family size and birth order. However, there were significant differences in socioeconomic status and educational level of both parents. Students with math difficulties predominantly came from families with lower professional categories and lower educational levels, while students without difficulties had parents in middle professional categories and higher educational levels. Overall, the findings indicate that family characteristics differ significantly between the two groups.

Table 10 Distribution of absolute and relative frequencies of students with and without difficulties in Mathematics in terms of the size of their families.

STUDENT CATEGORY	**FAMILY SIZE**		
	SMALL FAMILY	**LARGE FAMILY**	**TOTAL**
STUDENTS WITH DIFFICULTIES	**28 (60,9%)**	**18 (39,1%)**	**46 100%**
STUDENTS WITHOUT DIFFICULTIES	**19 (63,3%)**	**11 (36,7%)**	**30 100%**
TOTAL	**47 61,8%**	**29 38,2%**	**47 61,8%**

Statistical significance check: x^2= 0,047 df= 1 p=,829

Table 11Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of their order of birth

STUDENT CATEGORY	**BIRTH SERIES**			**TOTAL**
	FIRST BORN	**IN INTERMEDIATE**	**AFTERBORN**	
STUDENTS WITH DIFFICULTIES	**15 34,9%**	**7 16,3%**	**21 48,8%**	**43 100%**
STUDENTS WITHOUT DIFFICULTIES	**12 41,4%**	**6 20,7%**	**11 37,9%**	**29 100%**
TOTAL	**27 (37,5%)**	**13 (18,1%)**	**32 (44,4%)**	**72 (100%)**

Table 12Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in relation to the father's profession

STUDENT CATEGORY	PROFESSIONAL CATEGORY FATHER				TOTAL	
	LOW		MEDIUM			
STUDENTS WITH DIFFICULTIES	34 (72,3%)		13 (27,7%)		47 100%	
STUDENTS WITHOUT DIFFICULTIES	13 (43,3%)		17 (56,7%)		30 100%	
TOTAL	47	61,0%	30	39,0%	77	100%

Table 13 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in relation to the mother's profession.

STUDENT CATEGORY	PROFESSIONAL CATEGORY MOTHER				TOTAL	
	LOW		MEDIUM			
STUDENTS WITH DIFFICULTIES	37 (78,7%)		10 (21,3%)		47 100%	
STUDENTS WITHOUT DIFFICULTIES	16 (53,3%)		14 (46,7%)		30 100%	
TOTAL	53	68,8%	24	31,2%	77	100%

Table 14 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in terms of the educational level of the father.

STUDENT CATEGORY	FATHER'S EDUCATIONAL LEVEL				TOTAL
	PRIMARY SCHOOL	HIGH SCHOOL	LYCEUM	UNIVERSITY	
STUDENTS WITH DIFFICULTIES	21 (44,7%)	12 (25,5%)	10 (21,3%)	4 (8,5%)	47 (100,0%)
STUDENTS WITHOUT DIFFICULTIES	6 (20,0%)	7 (23,3%)	11 (36,7%)	6 (20,0%)	47 (100%)
TOTAL	27 (35,1%)	19 (24,7%)	21 (27,3%)	10 (13,0%)	77 (100%)

Table 15 Distribution of absolute and relative frequencies of students with and without learning difficulties in Mathematics in relation to the educational level of the mother.

STUDENT CATEGORY	MOTHER'S EDUCATIONAL LEVEL				TOTAL
	PRIMARY SCHOOL	HIGH SCHOOL	LYCEUM	UNIVERSITY	
STUDENTS WITH DIFFICULTIES	17 (36,2%)	13 (27,7%)	14 (29,8%)	3 (6,4%)	47 (100,0%)
STUDENTS WITHOUT DIFFICULTIES	3 (10,0%)	11 (36,7%)	10 (33,3%)	6 (20,0%)	30 (100%)

TOTAL	**20** **(26,0%)**	**24** **(31,2%)**	**24** **(31,2%)**	**9** **(11,7%)**	**77** **(100%)**

5.2.2.2 Comparisons of the students of the two groups in terms of perceptions and attitudes of their parents towards Mathematics

The study compared parental perceptions of the importance of mathematical ability and the functions of mathematics in two groups: students with difficulties and students without difficulties. Both groups ranked reading ability as most important, followed by mathematical ability and writing ability. Both groups recognized the necessity of mathematics for knowledge acquisition and professional application. The only notable difference was the order of preferences for second and third choices between the two groups.

5.2.2.3 Comparisons of students in both groups with their parents' assessments of their Math difficulties, hyperactivity and distraction of their children

Parents of students with learning difficulties in mathematics perceived their children to have moderate difficulty in the subject. Both groups of parents rated their children as moderately hyperactive. However, parents of students with disabilities reported concentration difficulties, while parents of students without difficulties did not. These assessments by parents are limited and do not constitute a diagnosis for ADHD. Professional evaluation is necessary.

5.2.2.4 Comparisons of the students of the two groups in terms of the level of ambition and in terms of the help they give to their children in their school work and especially in the work of mathematics.

The study compared parental involvement, study conditions, and assistance with homework between students with and without math difficulties. Parents of students without difficulties provided better study conditions and less homework assistance. Both groups valued education and preschool education equally. Parents of students with difficulties exhibited higher involvement and assistance with homework. However, there were no significant differences in school visits or the overall importance of education.

5.2.2.5 The difficulties in Mathematics and Language of the family members of the students of both groups

The study examined the presence of learning difficulties in family members of students with and without difficulties in mathematics. Although the observed differences were not statistically significant, a higher percentage of students with difficulties had family members who encountered difficulties in school compared to the group without difficulties. The types of difficulties varied, with mathematics being a common challenge for family members in both groups.

5.2.3 Comparisons of individual elements of the developmental history of the children of the two groups (with difficulties - without difficulties

The study examined various aspects of the development and early experiences of students with and without difficulties in mathematics. While there were some non-significant trends and slightly higher percentages of certain conditions or difficulties in the group with mathematics difficulties, there were no statistically significant differences between the two groups in most areas of development and early experiences examined, including pregnancy, birth and perinatal period, infancy development, preschool age, and school age.

5.2.4 Comparisons of the children of the two groups (with difficulties - without difficulties) in terms of their individual characteristics, preferences and interests

The study compared two groups: students with learning difficulties in mathematics and those without difficulties. The groups showed no significant differences in gender, game preferences, counting ability, and most personality traits. However, students with difficulties were younger on average and had lower interest in school lessons. There was a significant difference in the age distribution, with more students with difficulties falling into the "young" category. Both groups had similar preferences for games, prioritizing group games with outdoor activity, followed by quiet group games and games related to numbers and puzzles.

Table16 Distribution of absolute and relative frequencies of students with L.D. and W. D. in Mathematics as to the characteristic of adaptation to new situations

STUDENT CATEGORY	EASY ADAPTATION TO NEW CONDITIONS		TOTAL
	ΥΕΣ	NO	
STUDENTS WITH DIFFICULTIES	26 (56,5%)	20 (43,5%)	46 100%
STUDENTS WITHOUT DIFFICULTIES	22 (73,3%)	8 (26,7%)	30 100%

TOTAL	**48 63,2%**	**28 36,8%**	**76 100%**

Table 17 Distribution of absolute and relative frequencies of students with L.D. and W.D. in Mathematics as to whether he works carefully

STUDENT CATEGORY	**WORKS CAREFULLY**		**TOTAL**
	ΥΕΣ	**NO**	
STUDENTS WITH DIFFICULTIES	**16 (34,8%)**	**30 (65,2%)**	**46 100%**
STUDENTS WITHOUT DIFFICULTIES	**14 (46,7%)**	**16 (53,3%)**	**30 100%**
TOTAL	**30 39,5%**	**46 60,5%**	**76 100%**
Statistical significance check: F.E.T. ρ = ,343			

Table 18 Distribution of absolute and relative frequencies of students with L.D. and W.D. in Mathematics as to whether they make easy friendships+

STUDENT CATEGORY	**MAKES FRIENDSHIP EASY**		**TOTAL**
	ΥΕΣ	**NO**	
STUDENTS WITH DIFFICULTIES	**36 (78,3%)**	**10 (21,7%)**	**46 100%**
STUDENTS WITHOUT DIFFICULTIES	**20 (69,0%)**	**9 (31,0%)**	**29 100%**
TOTAL	**56 74,7%**	**19 25,3%**	**75 100%**
Statistical significance check: F.E.T. ρ = ,420			

Table 19 Distribution of absolute and relative frequencies of students with L.D. and W.D. in Mathematics as to whether they show persistence in what they are doing

STUDENT CATEGORY	**SHOWS PERSISTENCE WITH WHAT HE IS DEALING**		**TOTAL**
	ΥΕΣ	**NO**	
STUDENTS WITH DIFFICULTIES	**19 (41,3%)**	**27 (58,7%)**	**46 100%**
STUDENTS WITHOUT DIFFICULTIES	**18 (62,1%)**	**11 (37,9%)**	**29 100%**
TOTAL	**30 39,5%**	**46 60,5%**	**76 100%**
Statistical significance check: F.E.T. ρ = ,100			

5.3 STUDY OF EFFICIENCY OF EXPERIMENTAL TEACHING - COMPARISONS OF STUDENT PERFORMANCE OF EXPERIMENTAL GROUP AND OF CONTROL GROUP IN MATH

5.3.1 Introduction

The research study compares the performance of students with learning difficulties in mathematics between an experimental group that received teaching and a control group that did not. Teaching effectiveness is assessed by analysing differences in pretests and meta-tests, as well as post-tests and final tests. Mathematical abilities, intelligence quotients, and grades in mathematics and language courses are used as measures. The control group's performance in pretests and post-tests is also examined to understand the effect of standard teaching.

5.3.2 The equation of the experimental group and the control group

In the initial attempt to equate the experimental and control groups, slight discrepancies were found in pretest performance and mathematical abilities. The control group had higher average performance in mathematical abilities, while the experimental group showed better performance in general school and language areas. However, there were no significant differences in general intelligence or verbal and practical intelligence quotients. Overall, the groups were considered homogeneous in terms of performance and dispersions.

Table 20 The averages and the standard deviations of the quotient of general intelligence of the students of Experimental Group and Control Group. Checking the importance of the difference of the averages.

GROUPS	N	Avg.	StDev	STATISTICAL SIGNIFICANCE CHECK
Experimental	23	95,74	8,65	**t**= 0,071
Control	23	95,57	7,90	**p**= ,944
* Levene’s test for dispersion equality: F= 0,286, p= ,596				

Table 21The averages and the standard deviations of the verbal intelligence quotient of the students of Experimental Group and Control Group.. Checking the importance of the difference of the averages.

GROUPS	N	Avg.	StDev	STATISTICAL SIGNIFICANCE CHECK
Experimental	23	98,00	10,03	**t**= 0,407
Control	23	96,91	7,94	**p**= ,686
* Levene’s test for dispersion equality: F= 0,979 p= ,328				

Table 22The averages and the standard deviations of the quotient of practical intelligence of the students of Experimental Group and Control Group.. Checking the importance of the difference of the averages

GROUPS	N	Avg.	StDev	STATISTICAL SIGNIFICANCE CHECK
Experimental	23	94,13	9,35	**t**= -0,499
Control	23	95,48	8,96	p= ,620
* Levene's test for dispersion equality: F= 0,032 p= ,858				

5.3.3 Comparisons of the performance of the students of Experimental Group and Control Group immediately after the end of the teaching intervention

Significant differences were found between the experimental group and the control group in terms of their performance in the assessment of mathematical abilities. The experimental group showed higher performance in all individual tests of the assessment criterion, and these differences were statistically significant. The control group exhibited more heterogeneous performance with larger standard deviations. These findings support the effectiveness of the additional didactic support provided to the experimental group in improving their mathematical skills.

Table 23 The averages and the standard deviations of the answers of the students of Experimental Group and Control Group in the criterion of mathematical skills (metatest). Check the significance of the difference of the averages.

OVERALL PERFORMANCE IN METATEST	N	Avg.	StDev	STATISTICAL SIGNIFICANCE CHECK
Experimental	23	8,630	0,558	**t**= 10,864
Control	23	4,968	1,484	p= ,000
* Levene's test for dispersion equality: F= 17,127 p= ,000				

5.3.4 Comparisons of the performance of the students of Control Group before and immediately after the didactic intervention of the experimental group

The control group did not show significant improvement in their mathematical abilities compared to the pretest scores. Average performance remained relatively stagnant, with minor declines and improvements. The slight improvements were not substantial and may be attributed to individual students rather than significant progress for the entire group. Systematic teaching intervention is necessary for significant improvement, as the control group did not show notable progress without such intervention.

Table 24 The averages and the standard deviations of the answers of the students with learning difficulties in the Mathematics of Control Group in the criteria of mathematical skills (pretest and metatest). Check the significance of the difference of the averages.

CONTROL GROUP	N	Avg	StDev	STATISTICAL SIGNIFICANCE CHECK
PRETEST	22	4,559	1,207	t= -2,064
METATEST	22	4,968	1,484	p= ,052

5.3.5 Comparisons of the performance of the students of Experimental Group before and immediately after the teaching intervention

The teaching intervention in the experimental group led to a significant improvement in the students' performance in mathematical skills. The average performance reached levels comparable to students without learning difficulties in the pretest. The experimental group showed greater homogeneity in their performance compared to the control group. Improvement was observed in all individual tests, with significant differences in specific areas such as the position value of digits, numerical operations, and problem-solving. Overall, the systematic teaching approach was highly successful in enhancing the mathematical abilities of the experimental group.

5.3.6 Comparisons of the performance of the students of Experimental Group immediately after the teaching intervention and after six months

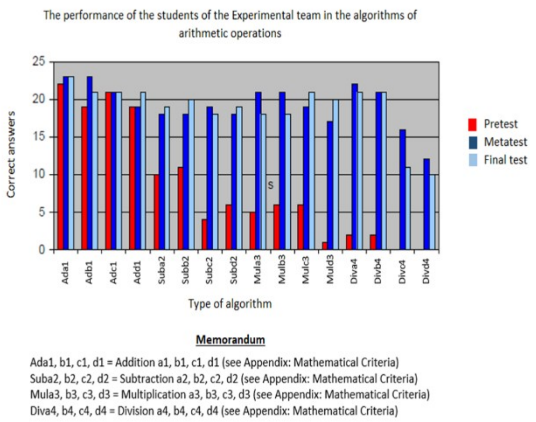
The learning improvements achieved in the experimental group were maintained after six months, with stable performance observed in the meta test and final test. Although there was a slight decrease in average performance and increased dispersion, the overall benefits of the intervention were maintained. Most individual tests showed no significant changes, indicating stable or improved performance.

5.3.7 The performance of the students of the two groups (Experimental and control) in the algorithms of the operations and in problem solving

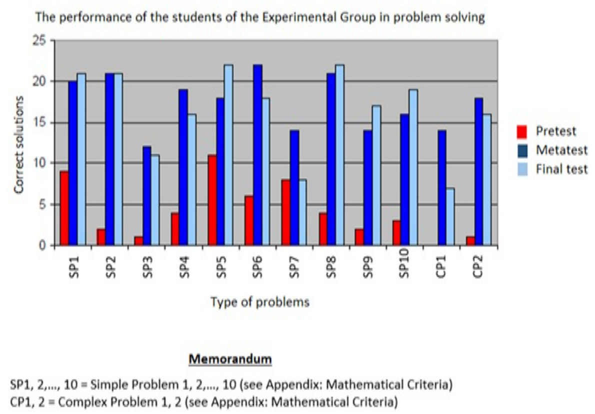
In this section we will examine the performance (correct answers) of students of both the experimental and the control group in areas of Mathematics that are of particular interest, such as algorithms of operations and problem solving. The correct answers of the students will be given diagrammatically with graphs, so that there is the possibility of comparing the performance of the students of the same group in the individual tests of the mathematical competence criteria. groups in the same tests.

5.3.7.1 The performance of the students of Experimental Group in the execution of algorithms of operations and in solving problems before and after teaching

The students in the Experimental Group initially struggled with mathematical algorithms, especially in subtraction, multiplication, and division, with division being the most challenging. However, after receiving systematic teaching, their performance significantly improved in all four operations, except for division with a two-digit divisor which remained difficult. This improvement was maintained six months later in the final test. In verbal problem-solving, the students initially had low performance, particularly in multiplicative structures. The teaching intervention helped improve their problem-solving skills, particularly in prosthetic problems related to combination, change, and simulation.



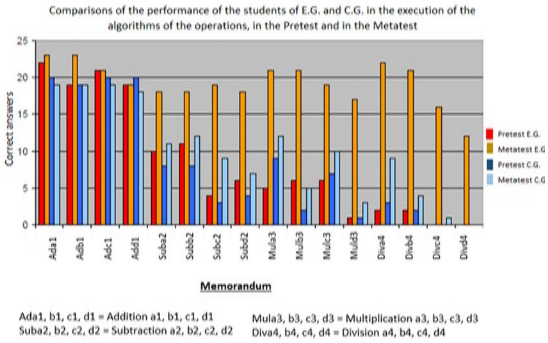
Graph 3 The performance of Experimental Group students in the algorithms of the four arithmetic



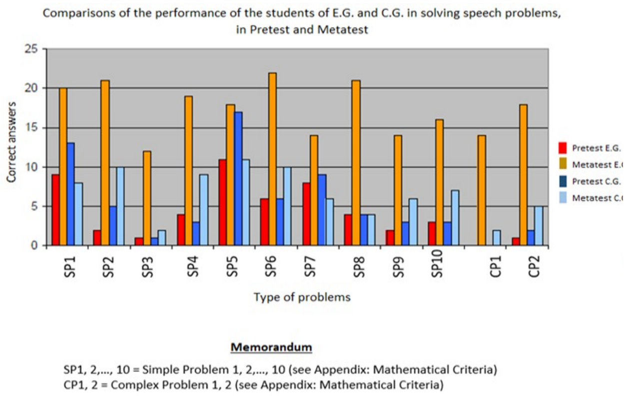
Graph 4The performance of the students of Experimental Group in solving speech problems

5.3.7.2 Comparisons of the performance of the students of Experimental Group and Control Group in the execution of algorithms of operations and in problem solving

The Experimental Group, who received systematic teaching, showed significant improvement in arithmetic operations, particularly in subtraction, multiplication, and division. In verbal problem-solving, the Experimental Group also demonstrated improvement after the teaching intervention. In contrast, the Control Group, who did not receive additional teaching support, had lower and stagnant performance in both arithmetic operations and verbal problem-solving. The results highlight the importance of systematic teaching for significant improvement in students with learning difficulties in Mathematics.



Graph 5The performance in the algorithms of the operations of the students of Experimental Group and Control Group before and after the didactic intervention that was made to the students of Experimental Group.



Graph 6The performance in solving the verbal problems of the students of Experimental Group and Control Group before and after the didactic intervention that was made to the students of Experimental Group

CONCLUSIONS & SUGGESTIONS

Conclusions

The research presented aims to understand the individual and family characteristics of students with learning disabilities in Mathematics and their classmates without difficulties. It also aims to design, implement, and evaluate a curriculum for teaching basic mathematical concepts and skills to such students. The research found the follow:

1. Students with LD showed particularly low performance in Arithmetic operations and Problem Solving.
2. Students with LD had the lowest achievement in Mathematics and the relatively highest in Reading, due to the fact that learning difficulties in Mathematics and Developmental Dyscalculia very often coexist with learning difficulties in language subjects and dyslexia.

3. Students with LD in Mathematics with normal intelligence, as well as the severity of the difficulties they face, is not correlated with their IQ.
4. Students with LD in Mathematics mainly come from families of low socio-economic level, while their WLD peers mainly belong to middle classes.
5. The mothers of students with LD mainly have a lower education, while those of WLD students mainly have medium or even higher education, and the parents of students of both groups consider reading ability to be of primary importance and mathematical ability secondarily.
6. Parents of students with LD report helping their children significantly more than parents of WLD students, both with their schoolwork in general and with math work.
7. Games related to numbers are the last preference, while group games with intense activity are ranked as the first preferences, the students of both groups do not differ significantly and they are just as social.
8. They also do not differ from each other in their personality characteristics, which is not the case with the degree of concentration in their schoolwork.

In terms of confirming the hypotheses, especially about the first hypothesis research has shown that differentiated instruction improves students' mathematical abilities because these often coexist with other learning difficulties, so differentiated instruction can be adapted to their needs. In addition, regarding the second hypothesis which is about students' specific cognitive profiles, the research showed that students with dyscalculia often struggle with basic number sense, such as understanding the quantity, magnitude, and relationships of numbers, exhibit weaknesses in working memory and have challenges to process multiple steps in math problems. Also, they have problems to organize multiple tasks. Finally, regarding the differences between individual and family characteristics of students with dyscalculia and those without mathematical learning difficulties, i.e. the third hypothesis is also confirmed because the research showed that students with LD in Mathematics mainly come from families of low socio-economic level, while their WLD peers mainly belong to middle classes. Also, the education level of the parents with children with LD differs from parents whose children do not have LD, while those do not help their children as the parents with children with LD.

Limitations of research

The survey findings have limitations due to factors such as the possibility of environmental factors affecting student performance, subjective evaluation in language lessons, and the lack of comprehensive neuropsychological evaluation tools.

Contributions of research

This research study on the implementation of the differentiated teaching approach to students with dyscalculia in the mathematics course can have several significant contributions to various aspects.

Dyscalculia is a learning disability that affects a student's ability to understand and process numerical and mathematical concepts. By studying the effectiveness of differentiated teaching strategies for students with dyscalculia, the research can contribute to making the education system more inclusive and accessible for all students, regardless of their learning differences. Also, the research can shed light on innovative and tailored teaching methods for students with dyscalculia. This could lead to the development of new educational tools, resources, and teaching practices that can benefit not only students with dyscalculia but also other learners who struggle with math concepts.

On the other hand, collaborative research involving educators, psychologists, mathematicians, and experts in special education can help establish a multidisciplinary approach to addressing learning disabilities like dyscalculia. This cooperation can lead to more comprehensive and holistic solutions in education and special needs support. Generally, effective differentiated teaching approaches can help students with dyscalculia feel more confident in their mathematical abilities and more motivated to engage in learning. Increased motivation can have positive effects on their overall academic performance and future educational and career prospects.

It is proven that students with dyscalculia can achieve significant success in an inclusive environment if a differentiated approach is applied; A toolkit for assessing factors and performance in students with dyscalculia (questionnaires) has been compiled, which can be used to collect data for such students.

Proposals for implementation

The teaching of Learning Disabilities in Mathematics and Psychodiagnostic Means of Learning Disabilities should be included in the curriculum of Pedagogical Departments and teacher training programs. Collaboration between educational institutions, associations, and publishers is needed to improve textbooks and teaching methods. Training programs and workshops for teachers and parents should be organized to raise awareness about learning difficulties in Mathematics. Schools should have small support classes with trained teachers, and their effectiveness should be evaluated.

Directions for future research

A comparative study between dyslexic students with and without dysnumeracy is recommended to identify factors influencing their performance in Mathematics. Assessments should focus on reading ability, spelling, and written expression. Developing valid tests for school performance and neuropsychological evaluations would aid in understanding learning disabilities. Investigating the impact of distraction on students with learning difficulties is necessary. Early diagnosis and research on developmental dysnumeracy from preschool age would contribute to effective treatment. Experimental research on effective teaching methods using educational technology should be conducted.

PUBLICATIONS

1. APPLICATION OF THE DIFFERENTIATED APPROACHTO THE MAINSTREAM MATHEMATICAL LEARNINGOF STUDENTS WITH DYSCALCULIA *Kosma Afroditi, PhD student*, *сборник „Образование и изкуства – традиции и перспективи“, Редактор: Замфиров, М. и кол. София, УИ „Св.Кл.Охридски“,с. 917-924, 2022, ISSN 2738-8999*
2. *ОБУЧЕНИЕ ПО МАТЕМАТИКА НА УЧЕНИЦИ С ДИСКАЛКУЛИЯ В ОБЩООБРАЗОВАТЕЛНА СРЕДА TEACHINIG MATHEMATICS STUDENTS WITH DYSCALCULIA IN THE MAINSTREAM EDUCATIONAL SETTINGSАфродити Козма, докторант, ФНОИ, СУ „Св. Климент Охридски“ Afroditi Kosma, с. 273- 277, ОБРАЗОВАНИЕ И ИЗКУСТВА: ТРАДИЦИИ И ПЕРСПЕКТИВИ Сборник доклади от Научно-практическа конференция, посветена на 80-годишнината от рождението на проф. д-р Георги Бижков, Редактор: Замфиров, М. и кол., София, УИ „Св.Кл. Охридски“, 2020г*
3. *ДИФЕРЕНЦИРАН ПОДХОД В ОБУЧЕНИЕТО ПО МАТЕМАТИКА НА ИНТЕГРИРАНИ УЧЕНИЦИ С ДИСКАЛКУЛИЯ. DIFFERENTIATED APPROACH*

IN MATHEMATICAL LEARNING OF MAINSTREAMED STUDENTS WITH DYSCALCULIAАфродити Козма, докторант, ФНОИ, СУ „Св. Климент Охридски“ Afroditi Kosta с. 266-272, ОБРАЗОВАНИЕ И ИЗКУСТВА: ТРАДИЦИИ И ПЕРСПЕКТИВИ Сборник доклади от Научно-практическа конференция, посветена на 80-годишнината от рождението на проф. д-р Георги Бижков, Редактор: Замфиров, М. и кол., София, УИ „Св.Кл. Охридски“, 2020г